

NEW IDEALS AND FILTER OF ON BG-ALGEBRA

Akeel Kareem Mohammad

Abstract: In this work as we introduce AK-ideal-BG algebra. AK-filter Also, we give some theorems and relationships among them.

INTRODUCTION

This concept is very important in the field of algebra and has great applications in the Internet and network science.

In (1966), Y. Imai and K. Is'eki introduced two classes of abstract algebras: BCK- algebras and BCI- algebras [3]. In (1999), J. Naggersand H. S.Kim introduced the notion of d-algebra, which is another use generalization of BCK-algebra [5]. In 1999 J. Naggers, Y. B. Junand H. S. Kim introduced the notion of d-ideal in d- algebra[6]. In (2008) Chang Bum Kim, Hee Sik Kim introduces the notion on BG-algebras ON BG-ALGEBRAS [4]. In (2012) C. A.Muhammad and. F.Ali, introduced multipliers in d-algebras [1]. This paper's objective is to introduce new types ideal and filter on BG-algebra its called AK- ideal and AK- filter. Also this paper studied the relationships between them.

PRELIMINARIES

Definition (2.1)[4]:

A BG-algebras a set X with a binary operation $*$ which satisfies the following axioms:

1. $x*x=e$
2. $x*e=x$
3. $(x*y)*(e*y)=x$, For all $x,y \in X$

Example(2.2)[4]

suppose $X=\{e,a,b\}$ be asset with the following table:

*	e	a	b
e	e	a	b
a	a	e	a
b	b	b	e

Then $(X,*,e)$ isa BG-algebra Not : (2.3)

In BG-algebra X , we denoted $e* X$ by $x*$ forever $yx \in X$.

Definition(2.4)

If $(X,*,e)$ be a BG-algebra and I be a non empty sub set of X . Then I

Is called an AK-ideal of X if for any $x,y \in X$,

- 1) $e \in I$
- (2) $x*y \in I$ and $y \in I$ imply $x \in I$.

Remark(2.5)

- 1- $\{e\}$ and X are AK-idealsof X . $\{e\}$ and X are called thetrivial idealsof x
- 2- An AK-ideal I of X issaid to be proper if $I \neq X$.

Example (2.6):

If $X = \{e, a, b, c\}$ be a set as shown in the subsequent table

*	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

Then $(X, *, e)$ is a BG-algebra .

$I_1 = \{e, a\}$, then I_1 is a KA-Ideal of X , but $I_2 = \{e, a, b\}$ is not ak-ideal of X , because $c * a = b \in I_2$, however $c \notin I_2$.

Proposition(2.7):

The intersection of a family of AK-ideals of a BG-algebra is AK-ideal.

Proof:

Let $\{I_i, i \in \Delta\}$ be a family of AK-ideals in BG-algebra X , so

$e \in I_i, \forall i \in \Delta$, then $e \in \bigcap_{i \in \Delta} I_i$.

Now, let $x * y \in \bigcap_{i \in \Delta} I_i, y \in \bigcap_{i \in \Delta} I_i$ then $x * y \in I_i, y \in I_i, \forall i \in \Delta$, since I_i is ideal, $\forall i \in \Delta$, then $x \in I_i, \forall i \in \Delta$. Thus $x \in \bigcap_{i \in \Delta} I_i$.

Hence $\bigcap_{i \in \Delta} I_i$ is AK-ideal.

Remark(2.8)

The union of two Ak-ideal in BG-algebra is not AK-ideal in general as shown in the following example.

Example(2.9):

Let $X = \{e, a, b, c\}$ and let $*$ a binary operation is defined on X by

		*	e	a	b	c
e	□	a	b	c		
□	□	e	c	b		
□	b	c	e	a		
□	□	a	a	e		

It is obvious that $(X, *, e)$ is a BG-algebra and $F_1 = \{e, a\}, F_2 = \{e, a\}$ are idel in X but $I_1 \cup I_2 = \{e, a, b\}$ is not a idel in X , since $(c * b) = a \in I_1 \cup I_2$, but $c \notin F_1 \cup F_2$.

PRELIMINARIES

In this portion, we provide definitions AK- filter in BG -algebra on x and study its relationships between them.

Definition(3.1)

A non empty subset F of a BG-algebra X is said to be AK-filter if.

1. $e \in F$
2. $(x * y) * z \in F, y \in F$ implies $x \in F$

Example(3.2)

Let $X = \{e, a, b, c\}$, is defined by

*	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

It is that $(X, *, e)$ is BG-algebra and $F_1 = \{e, a\}$ is AK-filter but

$F_2 = \{e, a, b\}$ is not AK- filter, since $(c * a) = b \in F_2$ but $c \notin F_2$

Proposition (3.3):

The inter section of a family of AK-filter in BG-algebra x is a AK-filter.

Proof:

Let $\{F_i, i \in \lambda\}$ be a family of AK-filters in BG-algebra X , so $e \in F_i, \forall i \in \lambda$ and $(x * y) * z \in F_i, y \in F_i$, then $(x * y) * z \in \bigcap_{i \in \lambda} F_i, y \in \bigcap_{i \in \lambda} F_i$, then $(x * y) * z \in F_i, y \in F_i, \forall i \in \lambda$. Since F_i is AK-filter, $\forall i \in \lambda$, it follows that $x \in F_i, \forall i \in \lambda$ and hence $x \in \bigcap_{i \in \lambda} F_i$ ■

Remark:(3.4)

In general the union of two Ak-filters is not filter on BG- algebra as shown in the following example.

Example(3.5):

Let $X = \{0, a, b, c\}$ and a binary operation $*$ is defined by

*	e	a	b	c
e	e	e	e	e
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

It is clear that $(X, *, e)$ is BG-algebra X and $F_1 = \{e, a\}, F_2 = \{e, b\}$ are AK-filters but $F_1 \cup F_2 = \{e, a, b\}$ is not AK-filter, since $(c * b)^* = a \in F_1 \cup F_2$, but $c \notin F_1 \cup F_2$

References:

[1] Chaudhry, Muhammad Anwar, and Faisal Ali. "Multipliers in α -algebras." *World Applied Sciences Journal* 18.11 (2012): 1649-1653.
 [2] Imai, Yasuyuki, and Kiyoshi Iseki. "On axiom systems of propositional calculi. I." *Proceedings of the Japan Academy* 41.6 (1965): 436-439.
 [3] K. Iseki, An algebra relation with Proposition Calculus Proc, Japan Acad, 42: pp. 26-29, 1966.
 [4] Kim, Chang Bum, and Hee Sik Kim. "On BG-algebras." *Demonstratio Mathematica* 41.3 (2008): 497-506.
 [5] Neggers, Joseph, Young Bae Jun, and Hee Sik Kim. "On d -ideals in d -algebras." *Mathematica Slovaca* 49.3 (1999): 243-251.
 [6] Neggers, Joseph, and Hee Sik Kim. "On d -algebras." *Mathematica Slovaca* 49.1 (1999): 19-26.